Exercise 53

Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = x + \frac{1}{x}, \quad [0.2, 4]$$

Solution

Take the derivative of the function.

$$f'(x) = \frac{d}{dx}\left(x + \frac{1}{x}\right)$$

$$= 1 - \frac{1}{x^2}$$

$$= \frac{x^2 - 1}{x^2}$$

Set what's in the numerator equal to zero, and set what's in the denominator equal to zero. Solve both equations for x.

$$x^2 - 1 = 0 \qquad\qquad\qquad x^2 = 0$$

$$(x+1)(x-1) = 0 \qquad\qquad\qquad x = 0$$

$$x = -1 \quad \text{or} \quad x = 1 \qquad\qquad\qquad x = 0$$

These are the critical numbers. Only $x = 1$ is within $[0.2, 4]$, so evaluate f here.

$$f(1) = 1 + \frac{1}{1} = 2 \qquad\qquad\qquad \text{(absolute minimum)}$$

Now evaluate the function at the endpoints of the interval.

$$f(0.2) = 0.2 + \frac{1}{0.2} = 5.2 \qquad\qquad\qquad \text{(absolute maximum)}$$

$$f(4) = 4 + \frac{1}{4} = 4.25$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $[0.2, 4]$.

The graph of the function below illustrates these results.